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## Geometric inequalities on Heisenberg groups

We establish geometric inequalities in the sub-Riemannian setting of the Heisenberg group  $\mathbb{H}^n$ . Our results include a natural sub-Riemannian version of the celebrated curvature-dimension condition of Lott-Villani and Sturm and also a geodesic version of the Borell-Brascamp-Lieb inequality akin to the one obtained by Cordero-Erausquin, McCann and Schmuckenschläger. The latter statement implies a sub-Riemannian version of the geodesic Brunn-Minkowski inequality. The proofs are based on optimal mass transportation and Riemannian approximation of  $\mathbb{H}^n$ . These results refute a general point of view, according to which no intrinsic geometric inequalities can be derived by using optimal mass transportation on singular spaces.