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"Symplectic decomposition, Darboux theorem and ellipticity"

Our first result concerns the classical Darboux theorem. We prove that if ω_m is the standard symplectic form and f is any symplectic form, then we can find a diffeomorphism φ, with optimal regularity, satisfying

$$\varphi^* (\omega_m) = f \quad \text{and} \quad \langle \nabla \varphi; \omega_m \rangle = 0$$

provided that f is a small perturbation of ω_m . Moreover we show that the above system is elliptic and that we have uniqueness, when coupled with a Dirichlet datum.

We then apply the above result to the so-called symplectic decomposition. We show that any map φ, satisfying appropriate assumptions, can be written as

$$\varphi = \chi \circ \psi$$

where

$$\langle \nabla \chi; \omega_m \rangle = 0, \quad \nabla \varphi + (\nabla \varphi)^t > 0 \quad \text{and} \quad \psi^* (\omega_m) = \omega_m \, .$$

The analogy with mass transportation and the Monge-Ampère equation, as well as with the polar decomposition, will be emphasized.

This is a joint work with Wifrid GANGBO and Olivier KNEUSS.