BOUNDARY BEHAVIOR OF BERGMAN-HARMONIC MAPS

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ABSTRACT. We discuss boundary regularity in the Dirichlet problem for the Bergman-harmonic map system, for maps $\Phi : \Omega \to S$ of a strictly pseudoconvex domain $\Omega \subset \mathbb{C}^n$ with the Bergman metric into a Riemannian manifold S. C^∞ regularity up to the boundary fails in general, as the Bergman Laplacian is elliptic in the interior of Ω but degenerates at the boundary. Smoothness up to $\partial\Omega$ of the solution Φ to the Dirichlet problem forces a compatibility condition, to be satisfied by the boundary values ϕ, which (under additional assumptions) is that ϕ be a subelliptic harmonic map.

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