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Measures with the mean value property for L-harmonic functions: an inverse problem

Let L be a linear hypoelliptic second order PDO in divergence form with nonnegative characteristic form in $\mathbb{R}^n$. A probability measure μ in an open set Ω is said to have the mean value property for the L-harmonic functions in Ω if there exists a point $x_0 \in \overline{\Omega}$ such that

$$u(x_0) = \int_\Omega u(y) d\mu(y)$$

for every nonnegative solution to $Lu = 0$ in Ω. In this case we will say that (Ω, μ, x_0) is an $L-$triple. In this talk we present general positive answers to the following inverse problem. Let (Ω, μ, x_0) and (D, ν, x_0) be $L-$triples. If $\mu = \nu$ in $\Omega \cap D$, is it true that $\Omega = D$ (and hence $\mu = \nu$) ?